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- There are **five problems** in this exam.
- There is **one bonus** problem.
- You have **80 minutes** to complete the exam
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Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	10	3	53

1. (**Short Questions**) Please answer each of the following five questions:
   (a) (2 points) Solve for x in the following equation:
   $$49 \cdot 7^{2x+7} = \frac{1}{7} \cdot 7^{3x+5}$$
   (b) (2 points) Mark the following statements as true or false.
   - (True/False) The derivative of the logarithm function $(\ln x)$ is the logarithm function $(\ln x)$.
   - (True/False) We can differentiate the expression
   $$f(x) = e^x \cdot e^{10x}$$
   without invoking the product rule.
   - (True/False) Every property of the logarithms we have discussed in class can be derived from the properties of the exponentials.

   (c) (2 points) Which of the following differential equations describe exponential decay? Choose all the correct answers.
   - $y' = -3y$
   - $y' = 0$
   - $y' = 3y$

   (d) (2 points) Is the following formula correct? Justify your answer.
   $$(f(x)g(x)h(x))' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$
   (e) (2 points) The function,
   $$f(t) = \frac{10}{1 + 9e^{-2t}},$$
   has a horizontal asymptote at $y = a$. Identify the value of a.

2. (10 points) (**Critical Points**) Find the critical points of the following function:
   $$f(x) = x^4 e^{-2x^2}$$

3. (**Exponential Decay**) The concentration of a certain pollutant in the atmosphere t hours after an industrial accident is $f(t) = te^{-2t}$.
   (a) (5 points) Is the concentration increasing or decreasing or neither when $t = 5$? Justify your answer using calculus.
   (b) (5 points) At what time is the concentration the largest? Confirm that you found a maximum.

4. (10 points) (**Calculus of Logs**) Consider the following function:
   $$f(x) = \ln(x(x^2 + 1)^{-2})$$
   Is $f(x)$ increasing or decreasing at $x = 1$? Show your work for full credit.
   Hint: First expand using laws for logarithms.

5. (**Exponential Models**) You invested \$10 at a certain annual rate of interest compounded continuously. After 4 years the investment was worth \$100.

(a) (5 points) What was the interest rate?

(b) (5 points) When will your investment be worth \$200?

6. (3 points (bonus)) (**Fun with Products**) Please answer the following bonus questions:

(a) (2 points) Let $f(x) = g(x)^n$ for some natural number $n \geq 1$. Use the product rule to derive the following formula:

$$f'(x) = ng(x)^{n-1}$$

In particular, if $g(x)$ is a polynomial function, then we have derived the generalized power rule for the case where the exponent is some natural number $n \geq 1$.

(b) (1 point) Consider the function

$$f(x) = \frac{g_1(x)}{g_2(x)}$$

Derive a formula for the derivative of $f(x)$. This is the quotient rule.

Hint: It is indeed possible to derive the formula for $f'(x)$ given our knowledge.

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1. (**Short Questions**) Please answer each of the following five questions:
   (a) (2 points) Solve for y in the following equation:
   $$(y^2)(y^4)^2(y^{-1})(y^{-2})^2 = 32$$
   (b) (2 points) Mark the following statements true or false:
      (a) There are infinitely many functions $f(x)$ such that $f'(x) = x$.
      (b) The following formula is correct:
      $$\int e^{-x^2} dx = -\frac{e^{-x^2}}{2x} + C$$
      (c) The following formula is correct:
      $$\int \ln x \, dx = x \ln x - x + C$$
   (c) (2 points) If $G(x)$ is an anti-derivative of $g(x)$, then write down an expression in terms of $G(x)$ that is equal to
   $$\int_0^5 g(x)\, dx$$
   (d) (2 points) Suppose someone picks a random real number x between 0 and 10. What is the probability that $2 \leq x \leq 7$?
   (e) (2 points) If $f(x, y)$ is a function of x and y, and you know that
   $$f(0,0) = 3, \quad \frac{\partial f}{\partial x}(0,0) = 2, \qquad \frac{\partial f}{\partial y}(0,0) = -5,$$
   estimate the value of $f(1, 1)$.
2. (10 points) (**Areas**) Find the area enclosed by the curves $y = 5x$ and $y = x^2 + 6$ between their points of intersection.
3. (**Probability**) Please answer the following two questions:
   (a) (5 points) Find λ so that
   $$f(x) = \lambda e^{-5x} \qquad 0 \leq x < +\infty$$
   is a probability density function.
   (b) (5 points) You study how long it takes students to finish an assignment. You find that it is described by the probability density function
   $$f(t) = \frac{1}{(1+t)^2} \qquad 0 \leq t < +\infty$$
   where t is the time measured in hours. What is the probability that a randomly chosen student takes between 0 and 5 hours to complete the assignment.
4. (**Integrals & Probability**) Please answer the following two questions:
   (a) (5 points) Evaluate the following integral:
   $$\int \left(\frac{1}{5x+1} + x^{1/2} + e^{2x} + x^{-2} \right) dx$$

(b) (5 points) When you flip a coin 10,000 times, the number of heads is approximately normally distributed with $\mu = 5000$ and $\sigma = 50$. What is the probability that the number of heads is between 4950 and 5075?
NOTE: Leave your answer in terms of values of $\Phi(x)$ for the standard normal distribution.

5. (**Partial Derivatives**) Please answer the following two questions:
   (a) (6 points) Consider the function
   $$f(x, y) = xe^{xy}$$
   Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial^2 f}{\partial x \partial y}$.
   (b) (4 points) Please answer the following two qualitative questions:
      (i). What does it mean for $f(x, y)$ to have a relative maximum at some point (x_0, y_0). Give an example of a function $f(x, y)$ that has a relative-maximum, and state the point (x_0, y_0) at which it has a relative maximum.
      (ii). What does it mean for $f(x, y)$ to have a saddle point at some point (x_0, y_0). Give an example of a function $f(x, y)$ that has a saddle point, and state the point (x_0, y_0) at which it has a saddle point.

6. (3 points (bonus)) (**Last Call**) Please answer the following bonus questions:
   (a) (1 point) In class, we have learnt how to differentiate and integrate functions of one variable. Examples of some functions we have analyzed:
   $$f_1(x) = x^2 + e^x \qquad f_2(x) = e^x + x^{1/2}$$
   We have also learnt how to compute partial derivatives of functions of two variables. Examples of some functions we have analyzed:
   $$g_1(x, y) = ye^{x^3} \qquad g_2(x, y) = x^2y^3 + e^x \ln(x^2y^2)$$
   Is it possible to *integrate* functions of *two* variables? If so, what do you think would be the method to approach this problem? Any thoughts based on your current understanding of calculus?
   (b) (1 point) We have discussed how to compute partial derivatives of a function of two variables. In principle, one can compute partial derivatives of a function of n-variables. Consider the following function of n-variables:
   $$f(x_1, \cdots, x_n) = \frac{e^{x_1}}{e^{x_1} + \cdots + e^{x_n}}$$
   Compute $\frac{\partial f}{\partial x_1}$.
   Note: Partial derivatives of such an n-variable function routinely pop up in the theoretical analysis of machine learning algorithms designed specifically to solve multi-classification problems.
   (c) (1 point) **Free bonus point!** Write down any interesting comments about the class, exams, bonus questions, or anything about the course. Your input is valuable; it is at least worth a point :)

Thanks for being part of the class this summer! Good luck with everything :)

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1. (**Short Questions**) Please answer each of the following five questions:
   (a) (2 points) Solve for x in the following equation:
   $$\frac{x^4 x^2}{x^{10}} = 16$$
   (b) (2 points) Fill in the blanks:
   "If a function has a local minimum at $x = x_0$, then the derivative of $f(x)$, denoted as $f'(x)$, switches from being _____ for $x < x_0$ to being _____ for $x > x_0$."
   (c) (2 points) If the revenue from selling x objects is $R(x)$ (measured in dollars), and we know that $R(20) = 1000$ and $R'(20) = 100$, estimate the value of $R(18)$.
   (d) (2 points) Which of the following functions (there may be more than one) has an inflection point? Choose all the correct answers.

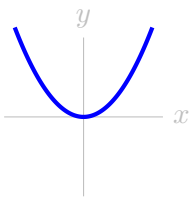


Figure 1: (a)

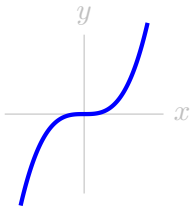


Figure 2: (b)

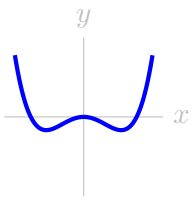


Figure 3: (c)

   (e) (2 points) Mark the following statements as true or false.
   - (True/False) The derivative of a non-vertical, straight line is a constant number.
   - (True/False) The tangent line to a function at a given point always intersects the function at exactly one point.
   - (True/False) A function with a constant rate of change will have a zero derivative everywhere.

2. (**Derivatives**) Consider the function $f(x) = (x^2 + 1)^4$.
   (a) (5 points) Compute the derivative $f'(x)$. You do not need to simplify your answer.
   (b) (5 points) Determine the equation of the tangent line to $y = f(x)$ at $x = 1$.

3. (**Curve Sketching**) Let $f(x) = x^4 - 4x^3$.
   (a) (5 points) Find the critical points of $f(x)$ and classify them as relative minima/maxima.
   (b) (2 points) Find the inflection points of $f(x)$.
   (c) (3 points) Sketch the graph of $f(x)$.

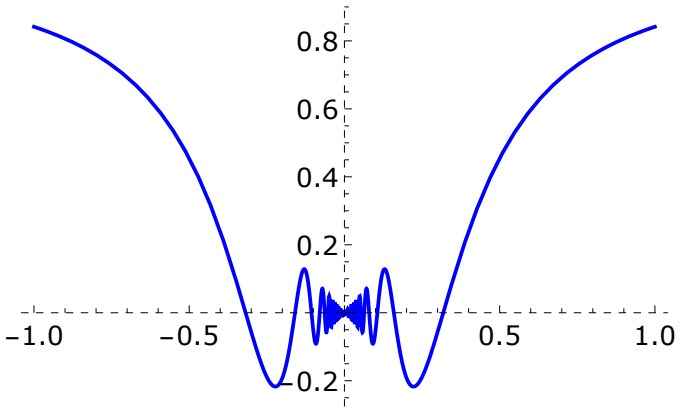
4. (**Kinematics with Calculus**) We throw a stone in the air, and its height after t seconds is described by the function
   $$h(t) = -16(t^2 - 2t - 3),$$
   measured in feet.
   (a) (5 points) Find the velocity of the ball when the stone hits the ground.
   (b) (5 points) Find the maximum height of the ball.

5. (10 points) (**Optimization with Calculus**) You want to build a closed rectangular box with square base, and with volume 24 cubic feet. The top costs 5 dollars per square foot, and the sides and base cost 1 dollar per square foot. What are the dimensions of the box that you can build for the minimum cost? (Give the length, width, and height as your answer.) Show the work that confirms that your answer is indeed a minimum.

6. (2 points (bonus)) (**ChatGPT vs. MATH 120**) Please answer the following bonus questions:

   (a) (1 point) How valuable do you find learning calculus, particularly in the era of ChatGPT (and AI more generally)? Is it worth the intellectual investment? Feel free to give your honest opinion and get a free point in return.

   (b) (1 point) Let's put your claim and ChatGPT to test. Consider the graph of a function below:



   I asked ChatGPT twice to compute $f'(0)$. It answered that $f'(0) = 0$ on the first try and that $f'(0)$ is undefined on the second try. What do you think happened? What do you think is the correct answer? Zero, undefined or something else? Would you be willing to amend your claim to (a) on the basis of this experiment? Why or why not?