

Please read the following instructions carefully:

- There are **eight problems** in this exam.
- There is **one bonus** problem.
- You have **90 minutes** to complete the exam
- The point distribution is given in the table below.
- Please write each solution on a separate page.
- You **must have your camera on** during the exam.
- This is a **closed book, closed notes exam**. You must not consult any resource while attempting the exam.
- Upload your work to Gradescope.
- Submitting the exam implies you abide by the honor pledge stated below:

I pledge on my honor that I have not given or received any unauthorized assistance on this quiz/examination

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	10	10	10	10	10	10	10	10	0	80

1. (10 points) A hemisphere-like shaped solid is generated by rotating the region above the x -axis, to the right of the y -axis and below the graph of the function $y = 4 - x^2$ around the y -axis. Compute the volume of the solid.
2. (10 points) The base of a solid, D , is a triangle in the xy plane with vertices $(0, 0)$, $(3, 0)$, $(0, 3)$. The cross sections perpendicular to the x -axis are semicircles. Write down the integral for the volume of the solid, D . **Do NOT evaluate the integral.**
3. (10 points) The parametric equation for the portion of the asteroid in the first quadrant is given by the parametric equations,

$$x(t) = R \cos^3 t, \quad y(t) = R \sin^3 t,$$

for t between 0 and $\pi/2$. Find the length of the portion of the asteroid.

4. (10 points) A hemispherical swimming pool has a radius of 6 feet. It is completely filled with water. Set up, **but DO NOT EVALUATE**, the integral for the work required to pump all of the water to a platform 5 foot above the top of the pool. **Place the origin at the TOP of the tank.** Assume the weight of water is 62.5 pounds per cubic foot.
5. (a) (5 points) Find the inverse function of the function $y = x - 2/x$ for $x > 0$.
(b) (5 points) Consider the function $f(x) = x^5 + x^3 + 1$. It can be shown that the function f has an inverse function. Noting that $f(1) = 3$, compute the derivative of the inverse function, $f^{-1}(y)$, at $y = 3$.

6. Solve the following questions:

- (a) (5 points) Find the derivative of the function,

$$f(x) = \log_5(e^x) + \sin^{-1}(x^2).$$

- (b) (5 points) Evaluate the integral,

$$\int 2x2^{x^2} dx$$

7. (10 points) Evaluate the integral,

$$\int \frac{dx}{x^2 + 3x + 3}.$$

8. Evaluate the following limits:

- (a) (5 points)

$$\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x^2}$$

- (b) (5 points)

$$\lim_{x \rightarrow \infty} \frac{\ln(e^x - 1)}{\ln(x)}.$$

9. (5 points (bonus)) Suppose you have three different algorithms for solving the same problem and each algorithm takes a number of steps that is of the order of one of the following functions:

$$n \log_2 n, \quad n^{\frac{3}{2}}, \quad n(\log_2 n)^2$$

Which of these functions gives an algorithm that is most efficient in the long run? Why? Justify your answer.