Please read the following instructions carefully:

- There are **five problems** in this exam.
- There is **one bonus** problem.
- You have **80 minutes** to complete the exam
- The point distribution is given in the table below.
- Please write each solution on a separate page.
- You **must have your camera on** during the exam.
- This is a **closed book, closed notes exam**. You must not consult any resource while attempting the exam.
- Upload your work to Gradescope.
- Submitting the exam implies you abide by the honor pledge stated below:

I pledge on my honor that I have not given or received any unauthorized assistance on this quiz/examination

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	10	3	53

1. (**Short Questions**) Please answer each of the following five questions:
   (a) (2 points) Solve for x in the following equation:
   $$49 \cdot 7^{2x+7} = \frac{1}{7} \cdot 7^{3x+5}$$
   (b) (2 points) Mark the following statements as true or false.
   - (True/False) The derivative of the logarithm function $(\ln x)$ is the logarithm function $(\ln x)$.
   - (True/False) We can differentiate the expression
   $$f(x) = e^x \cdot e^{10x}$$
   without invoking the product rule.
   - (True/False) Every property of the logarithms we have discussed in class can be derived from the properties of the exponentials.

   (c) (2 points) Which of the following differential equations describe exponential decay? Choose all the correct answers.
   - $y' = -3y$
   - $y' = 0$
   - $y' = 3y$

   (d) (2 points) Is the following formula correct? Justify your answer.
   $$(f(x)g(x)h(x))' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$
   (e) (2 points) The function,
   $$f(t) = \frac{10}{1 + 9e^{-2t}},$$
   has a horizontal asymptote at $y = a$. Identify the value of a.

2. (10 points) (**Critical Points**) Find the critical points of the following function:
   $$f(x) = x^4 e^{-2x^2}$$

3. (**Exponential Decay**) The concentration of a certain pollutant in the atmosphere t hours after an industrial accident is $f(t) = te^{-2t}$.
   (a) (5 points) Is the concentration increasing or decreasing or neither when $t = 5$? Justify your answer using calculus.
   (b) (5 points) At what time is the concentration the largest? Confirm that you found a maximum.

4. (10 points) (**Calculus of Logs**) Consider the following function:
   $$f(x) = \ln(x(x^2 + 1)^{-2})$$
   Is $f(x)$ increasing or decreasing at $x = 1$? Show your work for full credit.
   Hint: First expand using laws for logarithms.

5. (**Exponential Models**) You invested \$10 at a certain annual rate of interest compounded continuously. After 4 years the investment was worth \$100.
   (a) (5 points) What was the interest rate?
   (b) (5 points) When will your investment be worth \$200?

6. (3 points (bonus)) (**Fun with Products**) Please answer the following bonus questions:

   (a) (2 points) Let $f(x) = g(x)^n$ for some natural number $n \geq 1$. Use the product rule to derive the following formula:

   $$f'(x) = ng(x)^{n-1}$$

   In particular, if $g(x)$ is a polynomial function, then we have derived the generalized power rule for the case where the exponent is some natural number $n \geq 1$.

   (b) (1 point) Consider the function

   $$f(x) = \frac{g_1(x)}{g_2(x)}$$

   Derive a formula for the derivative of $f(x)$. This is the quotient rule.

   Hint: It is indeed possible to derive the formula for $f'(x)$ given our knowledge.