Please read the following instructions carefully:

- There are **five problems** in this exam.
- There is **one bonus** problem.
- You have **80 minutes** to complete the exam
- The point distribution is given in the table below.
- Please write each solution on a separate page.
- You **must have your camera on** during the exam.
- This is a **closed book, closed notes exam**. You must not consult any resource while attempting the exam.
- Upload your work to Gradescope.
- Submitting the exam implies you abide by the honor pledge stated below:

I pledge on my honor that I have not given or received any unauthorized assistance on this quiz/examination

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	10	3	53

1. (**Short Questions**) Please answer each of the following five questions:
   (a) (2 points) Solve for y in the following equation:
   $$(y^2)(y^4)^2(y^{-1})(y^{-2})^2 = 32$$
   (b) (2 points) Mark the following statements true or false:
      (a) There are infinitely many functions $f(x)$ such that $f'(x) = x$.
      (b) The following formula is correct:
      $$\int e^{-x^2}\,dx = -\frac{e^{-x^2}}{2x} + C$$
      (c) The following formula is correct:
      $$\int \ln x\;dx = x\ln x - x + C$$
   (c) (2 points) If $G(x)$ is an anti-derivative of $g(x)$, then write down an expression in terms of $G(x)$ that is equal to
   $$\int_0^5 g(x)\,dx$$
   (d) (2 points) Suppose someone picks a random real number x between 0 and 10. What is the probability that $2 \le x \le 7$?
   (e) (2 points) If $f(x,y)$ is a function of x and y, and you know that
   $$f(0,0) = 3, \quad \frac{\partial f}{\partial x}(0,0) = 2, \qquad \frac{\partial f}{\partial y}(0,0) = -5,$$
   estimate the value of $f(1,1)$.
2. (10 points) (**Areas**) Find the area enclosed by the curves $y = 5x$ and $y = x^2 + 6$ between their points of intersection.
3. (**Probability**) Please answer the following two questions:
   (a) (5 points) Find λ so that
   $$f(x) = \lambda e^{-5x} \qquad 0 \le x < +\infty$$
   is a probability density function.
   (b) (5 points) You study how long it takes students to finish an assignment. You find that it is described by the probability density function
   $$f(t) = \frac{1}{(1+t)^2} \qquad 0 \le t < +\infty$$
   where t is the time measured in hours. What is the probability that a randomly chosen student takes between 0 and 5 hours to complete the assignment.
4. (**Integrals & Probability**) Please answer the following two questions:
   (a) (5 points) Evaluate the following integral:
   $$\int \left(\frac{1}{5x+1} + x^{1/2} + e^{2x} + x^{-2}\right)\,dx$$

(b) (5 points) When you flip a coin 10,000 times, the number of heads is approximately normally distributed with $\mu = 5000$ and $\sigma = 50$. What is the probability that the number of heads is between 4950 and 5075?

NOTE: Leave your answer in terms of values of $\Phi(x)$ for the standard normal distribution.

5. (**Partial Derivatives**) Please answer the following two questions:

(a) (6 points) Consider the function

$$f(x, y) = xe^{xy}$$

Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial^2 f}{\partial x \partial y}$.

(b) (4 points) Please answer the following two qualitative questions:

(i). What does it mean for $f(x, y)$ to have a relative maximum at some point (x_0, y_0). Give an example of a function $f(x, y)$ that has a relative-maximum, and state the point (x_0, y_0) at which it has a relative maximum.

(ii). What does it mean for $f(x, y)$ to have a saddle point at some point (x_0, y_0). Give an example of a function $f(x, y)$ that has a saddle point, and state the point (x_0, y_0) at which it has a saddle point.

6. (3 points (bonus)) (**Last Call**) Please answer the following bonus questions:

(a) (1 point) In class, we have learnt how to differentiate and integrate functions of one variable. Examples of some functions we have analyzed:

$$f_1(x) = x^2 + e^x \qquad f_2(x) = e^x + x^{1/2}$$

We have also learnt how to compute partial derivatives of functions of two variables. Examples of some functions we have analyzed:

$$g_1(x, y) = ye^{x^3} \qquad g_2(x, y) = x^2y^3 + e^x \ln(x^2y^2)$$

Is it possible to *integrate* functions of *two* variables? If so, what do you think would be the method to approach this problem? Any thoughts based on your current understanding of calculus?

(b) (1 point) We have discussed how to compute partial derivatives of a function of two variables. In principle, one can compute partial derivatives of a function of n-variables. Consider the following function of n-variables:

$$f(x_1, \cdots, x_n) = \frac{e^{x_1}}{e^{x_1} + \cdots + e^{x_n}}$$

Compute $\frac{\partial f}{\partial x_1}$.

Note: Partial derivatives of such an n-variable function routinely pop up in the theoretical analysis of machine learning algorithms designed specifically to solve multi-classification problems.

(c) (1 point) **Free bonus point!** Write down any interesting comments about the class, exams, bonus questions, or anything about the course. Your input is valuable; it is at least worth a point :)

Thanks for being part of the class this summer. Good luck with everything!